Do I weigh less at the Equator than at the North Pole?

A person's mass (at any given time) is constant, it is a measure of the amount of matter in their body. However, their weight is not constant. It is a measure of the local force of gravity exerted on their mass. So while your mass stays constant whether you're standing on the Earth or on the moon, your weight will differ significantly between the two locations. The idea behind this article is that you must *weigh* less at the Equator compared to one of the poles, because although your mass is the same, and the force of gravity exerted on your body by the Earth is the same, your weight will differ due to the increased centrifugal forces exerted on your body at the Equator. At the North Pole, the rotation of the earth exerts no effect on your weight, whereas at the Equator, the spinning of the earth causes an inertial force to be exerted on your body, causing it to be pulled slightly away from the surface of the earth and reducing your weight. (Note that although centrifugal force is sometimes referred to as a fictitious force, it is only fictitious in the sense that it is due to rotation and not derived from gravity or another interaction between masses. Since it would not be present in a rotationally stationary system, it is considered inertial or fictitious, however (as anyone who has spent any time on a merry-go-round can tell you) it does exert a measurable force and therefore, would reduce your weight (though not your mass) in the system I am about to describe.) Finally, mass is usually measured in kilograms (kg) while weight is measured in Newtons (N: the standard scientific units) or more commonly, in pounds (lbs: the old imperial weight measure still in common use today).

To calculate how much someone's weight would be reduced at the equator compared to the north pole, well need to know a few things. We need:

1) the radius of the earth: 6,371 km (random sources on the internet seem to agree on this).
2) the period of one rotation: This is 24 hours, which is equal to 24(60)(60)=86400 seconds.

Force equals mass times acceleration, as in Newton's second law shown below.

$$F = ma$$

On the surface of the earth, the standard value taken for acceleration is 9.8 m/s^2. For a person massing 80 kg then, their weight (the amount of force exerted on their body by the Earth due to gravity) would be:

$$F = (80)(9.8) = 784\ N$$

Here we use Newtons (N) as the measurement of weight, we could just has easily have used pounds (lbs), although a different acceleration constant would have to be used. In this case, a person massing 80 kg would weigh the following:

$$F = (80)(2.2) = 176\ lbs$$

So this value, 176 lbs, is the weight of an 80 kg person when he or she is standing on the surface of the Earth. For simplicity's sake I am assuming that the radius of the earth doesn't change between the North pole and the Equator (untrue, since the earth actually bulges out at the Equator, but I can leave that modification calculation for another day). This value is what a person should weigh at the North pole. However, at the Equator, centrifugal force should act to reduce this weight. We can calculate how much by starting the analysis below.

Force is again equal to the produce of mass and acceleration, but in a rotating frame, acceleration is different. We are now referring to the centrifugal acceleration pulling a body away from the surface of the earth, and not the acceleration due to the Earth's gravity. I'll use the subscript c to denote centrifugal acceleration a_c. (Note to purists: technically, a_c is referring to *centripetal* force, which is the central force keeping the object's motion in a circle. However, a detailed explanation of the differences between the centripetal and centrifugal concepts is beyond the scope of this article, and would probably only confuse everyone, including me. Please be forgiving...)

The standard equation that governs this situation is as follows:

$$F = ma_c = m\frac{v^2}{r}$$

where v is the tangential velocity in m/s and r is the radius of curvature (in this case the distance to the center of the earth). We need to calculate the tangential velocity v.

Now we know that the distance to the center of the earth is 6371000 meters, and that the Earth rotates once around its axis every 24 hours or 86400 seconds. To find the velocity of a body sitting on the surface of the Earth, we need to know how far it would travel in a given time. We can use the following simple formula to find the circumference of the Earth at the Equator.

$$C = 2(\pi)r = 2(3.14)6371000 = 40026189.8 \text{ m}$$

Knowing this distance, and knowing that a person standing on the Equator travels this distance in 86400 seconds (the time of one rotation of the earth, i.e. one day), we can easily calculate the velocity.

$$v = \frac{d}{t} = \frac{40026189.8 \text{ m}}{86400 \text{ s}} = 463.27 \text{ m/s}$$

So a hypothetical person standing at the Equator is travelling at a speed of nearly 500 m/s (with respect to the center of the earth). We can now plug this value into the previous equation to determine the centrifugal force pulling this person away from the center of the earth. Let's take the mass to again be 80 kg.

$$F = ma_c = m\frac{v^2}{r} = (80)\frac{463.27^2}{6371000} = 2.695 \text{ N}$$

Taking this as a percentage of the previously calculated weight of 784 N, we can figure out that 2.695 N is only 0.344%. Therefore, at the equator, a person of mass 80 kg would have an *apparent* decrease in mass of (0.00344)(80)= 0.2752 kg. In more common terms, this person (who as you recall, has a *weight* of 176 lbs), would have a decrease in weight of (0.00344)(176)= 0.6054 lbs.

A shortcut equation of getting from knowing the radius and period of rotation to finding the weight decrease is to use the following equations.

$$F = ma = mr\frac{4\pi^2}{T^2} \text{ where T is the period of rotation in seconds}$$

Using the above values and substituting in, we have:

$$F = mr\frac{4\pi^2}{T^2} = (80)(6371000)\frac{4(3.14)^2}{86400^2} = 2.695 \text{ N}$$

which, as you can see, is the same result as above.

A generalized formula for the decrease in weight as a percentage according to latitude is as follows:

$$weight\ decrease = cos(\theta) * (0.344)\ \%$$

where θ is the latitude in degrees.

So for someone living on the 49th parallel (the border between Canada and the US in the west), their percentage weight decrease would be the following:

$$weight\ decrease\ (\%) = cos(49) * (0.344) = 0.226\ \%$$

And the weight decrease for a person weighing 176 lbs living at this latitude would be:

$$weight\ decrease\ (lbs) = (0.00226) * (176) = 0.3972\ lbs$$

In conclusion, a person living anywhere on Earth (except at the North or South Pole) will experience a weight decrease due to the centrifugal force generated by the rotation of the Earth. This decrease is too small to be measured by most household scales, so don't bother heading to central latitudes before seeing if the latest diet has really taken off the pounds.